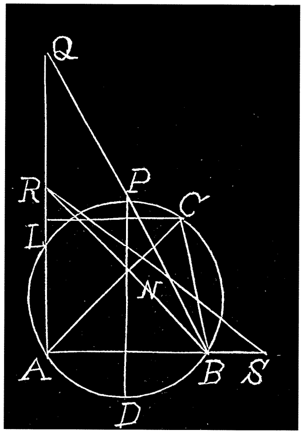


We further assume that $p>a<a\cot\frac{1}{2}C$. The solution would be just as simple with any other possible assumption.

On AB describe the segment ACB containing the given angle C. Draw the diameter PD perpendicular to AB. Also draw AQ perpendicular to AB. Draw BP meeting AQ in Q. With B as a center and a radius equal to p, describe an arc cutting AQ in R. With R as a center and a radius equal to $AQ-p$, describe an arc cutting AB produced in S. On SR measure off $SN=SA$. Then RN=altitude required. Take $AL=RN$, and draw CL parallel to AB, cutting the circle in C. Draw AC, BC. Then ACB is the required triangle. Let x, y, z be the sides BC, AC, and the altitude. Then $xy\sin C=az$, $x+y-z=p$, $a^2=x^2+y^2-2xy\cos C$.

$\therefore a^2+2xy(1+\cos C)=(p+z)^2$.

$\therefore z^2-2(a\cot\frac{1}{2}C-p)z=a^2-p^2$.

$\therefore z=a\cot\frac{1}{2}C-p-\sqrt{[(a\cot\frac{1}{2}C-p)^2-(p^2-a^2)]}$.

$\angle AQB=\frac{1}{2}C$, $AQ=a\cot\frac{1}{2}C$, $RS=a\cot\frac{1}{2}C-p$, $AR=\sqrt{[p^2-a^2]}$, $AS=\sqrt{[(a\cot\frac{1}{2}C-p)^2-(p^2-a^2)]}$.

$\therefore RN=z$. $\therefore$ The triangle ACB contains all the required parts.

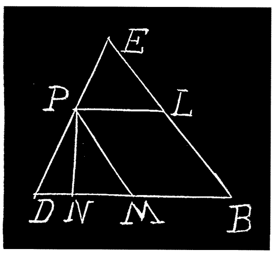
Also solved by J. Scheffer.

364. Proposed by R. C. ARCHIBALD, Providence, R. I.

Between the side of a given rhombus and its adjacent side produced, to insert a straight line of a given length and directed to the opposite corner. ["Euclidean constructions" are particularly desired.]

Solution by C. N. SCHMALL, New York City, and J. SCHEFFER, A. M., Hagerstown, Md.

This construction cannot be effected by Euclidean geometry. This will be evident from the algebraic analysis of the conditions.



Let $MBLP$ be the given rhombus, and P the given corner. Also let DE be the required line, so that $DE=l$=given length. On AB drop the perpendicular PN. Now let $MB=BL=a$, $MN=c$, $DM=x$. Then we have, in $\triangle DMP$,

$$DP^2=DM^2+MP^2-2DM.MN=x^2+a^2-2cx.$$

Also, $DM^2:DP^2=DB^2:DE^2$;

that is, $x^2:(x^2+a^2-2cx)=(x+a)^2:d^2$.

$$\therefore d^2=\frac{(x+a)^2(x^2-2cx+a^2)}{x^2}=(x+a)^2\left(1-\frac{2c}{x}+\frac{a^2}{x^2}\right)$$

$$=x^2+2(a-c)x+2a(a-2c)+2a^2(a-c).\frac{1}{x}+\frac{a^4}{x^2}.$$

Also solved by G. B. M. Zerr.

CALCULUS.

286. Proposed by R. D. CARMICHAEL, Princeton University.

Solve the differential equation

$$\begin{aligned}[a_0x^3+a_1x^2y+a_2xy^2+(a_0-a_1+a_2)y^3\\ +a_3x^2+a_4xy+a_5y^2+a_6x+a_7y+a_8]dx\\ +[a_0y^3+a_1xy^2+a_2x^2y+(a_0-a_1+a_2)x^3\\ +a_3y^2+a_4xy+a_5x^2+a_6y+a_7x+a_8]dy=0.\end{aligned}$$

I. Solution by W. W. BEMAN, Prosessor of Mathematics, University of Michigan, Ann Arbor, Mich.

Putting $x=s+t$, $y=s-t$, and afterward $t^2=w$, the equation takes the linear form

$$\frac{dw}{ds}+\frac{4(3a_0-2a_1+a_2)s+2(a_3-a_4+a_5)}{4(a_1-a_2)s^2+2(a_3-a_5)s+a_6-a_7}.w$$

$$=-\frac{4(a_0+a_2)s^3+2(a_3+a_4+a_5)s^2+2(a_6+a_7)+2a_8}{4(a_1-a_2)s^2+2(a_3-a_5)s+a_6-a_7}.$$

Or, putting $x+y=2u$, $xy=2v$, the equation takes the linear form

$$\frac{dv}{du}+\frac{4(3a_0-2a_1+a_2)u+2(a_3-a_4+a_5)}{4(a_1-a_2)u^2+2(a_3-a_5)u+a_6-a_7}.v$$

$$=\frac{8a_0u^3+4a_3u^2+2a_6u+a_8}{4(a_1-a_2)u^2+2(a_3-a_5)u+a_6-a_7}.$$

From each of these solutions it is obvious that the original equation has an integrating factor of the form $f(x+y)$.